Q) Evaluate. $\int \frac{x \cdot \sin^{-1}x}{\sqrt{1-x^2}}\, dx$

Solution, Given, $\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}}\, dx$

Now put $x = \tan^{-1} y$, $y = \tan^{-1} x$

put $\sin^{-1}x = y$

$\therefore x = \sin y$

$$\cancel{= \int \frac{\sin y \cdot y}{\sqrt{1-\sin^2 y}}}$$

$$\cancel{= \int \frac{y \sin y}{\cos y} = y \tan y}$$

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$\therefore y = \sin^{-1}x$

$\sin y = \frac{x}{\sqrt{1+x^2}}$

$\sin y = \frac{x}{1}$

$h = \sqrt{1-x^2}$

$\cos y = \sqrt{\ }$

Q: $\sin^{-1}x = y$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$

$$\therefore dx = \sqrt{1-x^2}\,dy$$

$\therefore$

$$= \int \frac{\sin y \cdot y}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\,dy$$

$$= \int y \cdot \sin y \cdot dy$$

$$= y \cdot (-\cos y) + \sin y$$

$$= -y\cos y + \sin y$$

$$= -\sin^{-1}x \cdot \cos(\sin^{-1}x) + \sin \sin^{-1}x$$

$$= -\sin^{-1}x \cdot \cos(\sin^{-1}x) + x$$

$$= -\sin^{-1}x \cdot \cos y + x$$

$\therefore$ $y = \sin^{-1}x$

$x = \sin y$

$\sin y = \frac{x}{1}$

$\cos y = \frac{\sqrt{1-x^2}}{1}$

$\therefore y = \cos^{-1}\sqrt{1-x^2}$

On putting the above values in the equation we get —

$$= -\sin^{-1}x \cdot \cos \cdot \cos^{-1}\sqrt{1-x^2} + x$$

$$= -\sin^{-1}x \cdot \sqrt{1-x^2} + x$$

$\therefore y = -\sqrt{1-x^2}\cdot \sin^{-1}x + x$ Ans

Q) Evaluate $\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\,dx$.

Solution: Given,

$$\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\,dx.$$

Let $\sin^{-1}x = y$, $\sin y = x$, $\cos y = \frac{dx}{dy}$

then, $dx = \cos y\, dy$

$$= \int \frac{y}{(1-\sin^2 y)^{3/2}}\cos y\, dy$$

$$= \frac{y}{(\cos^2 y)^{3/2}}\cos y\, dy$$

$$= \frac{y}{\cos^{\cancel{3}} y}\cancel{\cos y}\, dy$$

$$= \int \frac{y}{\cos^2 y}\,dy.$$

$$= \int y \sec^2 y\, dy$$

$$= y\cdot \tan y - \int \tan y$$

$$= y\cdot \tan y - \log \sec y \quad [\because \int \tan y = \log \sec x]$$

$= \cancel{\sin^{-1}x \cdot \tan \sin^{-1}x = \log \sec \sin^{-1}x}$

$=$

$= \cancel{\sin^{-1}x \cdot \tan^{-1}}$

$=$

$\therefore y = \sin^{-1}x$ $\quad = \sin^{-1}x\cdot\frac{x}{\sqrt{1-x^2}} = \log\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\therefore \frac{x}{1} = \sin y$ $\quad = \sin^{-1}x\cdot \frac{x}{\sqrt{1-x^2}} - \log(1) - \log\sqrt{1-x^2}$

$\tan y = \frac{x}{\sqrt{1-x^2}}$

$\cos y = \frac{\sqrt{1-x^2}}{1}$ $\quad = \sin^{-1}x\frac{x}{\sqrt{1-x^2}} + \log\left(\sqrt{1-x^2}\right)$ Ans

Q.17) Evaluate $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$

Solution: Given $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Now, put $x = \cos\theta$.

$1 = -\sin\theta \frac{d\theta}{dx}$

$\therefore dx = -\sin\theta$

$= \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \times -\sin\theta$

$= \int \tan^{-1}\sqrt{\frac{\cancel{2}\cdot\sin^2\theta/2}{\cancel{2}\cdot\cos^2\theta/2}}$ $[\because 2\sin^2\theta/2 = 1-\cos\theta]$

$= \int \tan^{-1}\sqrt{\tan^2\theta/2}$

$= \int \tan^{-1}\tan\theta/2\, d\theta \times -\sin\theta$

$= -\theta/2 \sin\theta\, d\theta$

Now using ILATE we get

$-\frac{1}{2}\left[-\theta\cos\theta + \int 1 \cos\theta\right.$

$= -\frac{1}{2}\left[-\theta\cdot\cos\theta + \sin\theta\right]$

$= -\frac{1}{2}\left[-\theta\cos\theta + \sin\theta\right]$

$= \frac{1}{2}\theta\cos\theta - \frac{1}{2}\sin\theta$

~~$\theta/2$ $\cos$~~

$\cos\theta = x$ $b = x$, $h = 1$, $p = \sqrt{1-x^2}$

$= \frac{1}{2}\cos^{-1}x\cdot\cancel{\cos x} - \frac{1}{2}\sqrt{1-x^2}$

~~$\cos\theta$~~: $\sin\theta = \frac{\sqrt{1-x^2}}{1}$ $\Rightarrow \frac{1}{2}\cos^{-1}x\cdot x - \frac{1}{2}\sqrt{1-x^2}$

Ans